90642

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Statistics and Modelling, 2004

90642 Calculate confidence intervals for population parameters

Credits: Three
9.30 am Monday 15 November 2004

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–STATF.

You should answer ALL the questions in this booklet.

If you need more space for any answer, use the pages provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–7 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement Criteria	*For Assessor's use only*				
Achievement		**Achievement with Merit**		**Achievement with Excellence**	
Calculate confidence intervals for population parameters.		Demonstrate an understanding of confidence intervals.		Analyse estimates of population parameters.	
Overall Level of Performance					

You are advised to spend 35 minutes answering the questions in this booklet.

Show **ALL** working.

QUESTION ONE

A researcher is investigating the effects of lake fishing in a region of the country. Lake Ackland, Lake Brightwater and Lake Crystal are three lakes in the region.

The researcher is studying the lengths of fish that were caught, the weights of fish that were caught and the number of fish that were caught by anglers in the last season.

(a)   A random sample of 50 fish was taken from anglers who fished Lake Ackland in the last season. When these fish were weighed it was found that the mean weight was 1283 grams and the standard deviation was 316 grams.

Find a 95% confidence interval for the mean weight of all fish that were caught by Lake Ackland anglers in the last season.

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(b)   A random sample of Lake Ackland anglers was interviewed. Of the 120 anglers interviewed, 23 said that they return to the lake all fish that they catch.

Find a 95% confidence interval for the proportion of all Lake Ackland anglers who return to the lake all fish that they catch.

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(c) Each of the 120 Lake Ackland anglers in part (b) were asked how many fish they had caught in the last season. The resulting data had a mean of 15.85 and a standard deviation of 8.45.

Find a 99% confidence interval for the mean number of fish caught by all Lake Ackland anglers in the last season.

(d) The researcher wants his estimate of the mean length of fish that were caught in Lake Ackland in the last season to be within 20 millimetres of the true mean length, with 95% confidence. Previous research has found that the standard deviation of lengths of fish that are caught in Lake Ackland is 80 millimetres.

What is the **minimum** sample size that is required to meet this condition?

QUESTION TWO

Anglers have suggested that there was no difference between the mean number of fish caught per angler for all anglers fishing in Lake Brightwater in the last season, and the mean number of fish caught per angler for all anglers fishing in Lake Crystal in the last season.

In order to investigate this claim the researcher gathered the following data:

	Lake Brightwater	Lake Crystal
Sample size	80	95
Sample mean	17.65	15.40
Sample standard deviation	6.53	7.66

Let μ_1 be the mean number of fish caught per angler for all anglers fishing in Lake Brightwater and μ_2 be the mean number of fish caught per angler for all anglers fishing in Lake Crystal in the last season.

(a) Find a 95% confidence interval for $\mu_1 - \mu_2$.

(b) Do the researcher's results suggest that there is a difference between μ_1 and μ_2?

Use statistical reasoning to justify your answer.

QUESTION THREE

The researcher completed a pilot survey using a sample of 30 fish that had been caught in Lake Crystal.

He calculated a 99% confidence interval for μ, the mean length (in millimetres) of fish caught, to be $414 < \mu < 486$.

(a) What sample size would be needed in his final survey if the researcher wants the 99% confidence interval to be one-third ($\frac{1}{3}$) of the width of the confidence interval obtained in the pilot survey?

Assume that the standard deviation of fish length determined in the pilot survey will also apply to the final survey.

(b) In part (a) above, the researcher wanted the width of the final confidence interval to be one-third ($\frac{1}{3}$) of the width of the confidence interval that had been obtained in the pilot survey.

Suppose that, in general, the researcher wants the width of a confidence interval to be $\frac{1}{k}$ of the length of the 99% confidence interval that had been obtained using a pilot survey with a sample of size 30 (k is a positive integer).

Obtain a formula that would give the required sample size (n) for the final survey. Give mathematical reasons for your answer.

Extra paper for continuation of answers if required.
Clearly number the question.

Assessor's use only

Question number

**Extra paper for continuation of answers if required.
Clearly number the question.**

Question
number